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## The Case for using semi-Lagrangian advection in the semi-Implicit integration scheme for the Unified Model

by

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# The Case for using semi-Lagrangian advection in the semi-Implicit integration scheme for the Unified Model.

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## 1 Purpose of this document

The purpose of this document is to review the advantages and disadvantages of semi-Lagrangian advection and to propose that because of the significant advantages over Eulerian advection methods only the semi-Lagrangian version of the proposed ' semi-Implicit Integration Scheme for the Unified Model', Cullen et al [5] is developed. Details of the semi-Lagrangian algorithm for this proposed integration scheme can be found in Mawson [12]. For a detailed review of semi-Lagrangian advection we refer the reader to the paper of Staniforth and Côté [17].

## 2 Summary of semi-Lagrangian versus Eulerian advection schemes

**Computational stability** The Eulerian methods in use in the current Unified Model are *conditionally stable*, so for given wind speed and grid-length there is a limit on the timestep that can be taken. The semi-Lagrangian method is *unconditionally stable* and the only restriction on the timestep is that imposed by the physics of the problem to be solved.

**spatial accuracy** Eulerian finite-differences on a unequally spaced grid are only first order accurate. Semi-Lagrangian advection is trivially implemented to an arbitrary order of accuracy on unequally spaced grids. In atmospheric models the benefit of this should be much better vertical advection of quantities which are highly non-linear, such as water vapour around the tropopause and potential temperature. However, this benefit has so far not been obtained by either ECMWF or NCAR, in fact worse performance has been found. See Section 2 for further discussion of this point.

**temporal accuracy** No significant differences for advection.

**pole problems** The pole has been a source of problems in almost every finite-difference model ever developed on the sphere and people often refer to the '*pole problem*' as one reason for not using this type of model. The semi-Lagrangian advection scheme does not suffer from any pole problems for advection of a scalar and when coupled with the vector approach for momentum, see Bates et al [1], provides a neat solution to all pole problems.

**phase errors** The semi-Lagrangian algorithm does not suffer from any phase error. The Eulerian algorithms used in the Unified Model suffer from severe phase errors as the flow direction becomes cross-polar. To show the effects of the pole we refer the reader to figures 1, 2 and 3. These are results from a test problem of advection of a cosine bell on the sphere under solid body rotation, see Malcolm [11] and Mawson [13] for more details. The bell is initially on the equator, figure 1, and the flow takes it directly over the pole. Figure 2 shows the result after 6 days with the tracer advection scheme when the bell should have completed half a revolution and have returned to the equator. In the figure it is easily seen that rather than being at the equator the bell is at 45 north, and hence moving at only three-quarters of the true velocity. Also the bell which initially was symmetrical has undergone quite severe distortion. Figure 3 shows the result for the positive definite version of the semi-Lagrangian algorithm after a complete revolution. There is no phase error and only a slight distortion of the bell in the direction of the flow. The solution for the current Unified Model advection scheme is not shown as it is significantly worse than the tracer advection scheme.

**Positive Definiteness** This can be included in the Eulerian scheme only by using an approach like the tracer advection scheme, which uses successive applications of one-dimensional flux limiters to achieve positive definiteness, see Cullen [4]. The search for a multi-dimensional limiter is still proceeding in the mathematical world with limited success. The inclusion of positive definiteness in the semi-Lagrangian scheme is simply a matter of ensuring that the interpolation scheme is positive definite. This can be done by

using a formally positive definite interpolation, see for example Williamson and Rasch [18], or by simply constraining the solution to lie between the values that form the grid-box surrounding it, see Bermejo and Staniforth [3], Priestley [15].

**Conservation** It is a relatively easy task to make Eulerian methods conservative, although the difficulty increases as the spatial and temporal accuracy is increased. Semi-Lagrangian methods are not formally conservative, although integrations with them have shown that the non-conservation is small, see for example Moorthi et al [14], and that the integral of the quantity being advected tends to oscillate about its original value with very small amplitude. The method can be made conservative by simply adding a constant on to all the values to restore the integral to its original value. Priestley [15] has developed a more complicated approach which restores conservation by changing the values at the smallest number of points possible and tries to use just the points where the interpolation error is largest. This method has been implemented in a shallow-water model by Gravel and Staniforth [8] and they found no noticeable effect of the procedure on the solution obtained. Solutions for conservative and non-conservative versions were very similar. This procedure has also been used successfully by Garcia-Navarro and Priestley [6]. Recently Leslie and Purser [10] have proposed a cascade interpolation procedure which enables conservation to be obtained without recourse to an a posteriori method. It is yet to be shown that this interpolation approach is applicable to the proposed New Unified Model integration scheme.

# 3 Experience of semi-Lagrangian advection in other models.

Most operational numerical weather prediction centres are using semi-Lagrangian advection methods or will be doing so in the near future. ECMWF are using a three time-level semi-Lagrangian scheme but are intending to go to a two time-level scheme to obtain the inherent factor of two cost saving gained from using the two-time level version over the three. They could not afford to run their current

T213 resolution with their Eulerian integration scheme because of the timestep limit it imposed, whilst they are able to run T213 semi-Lagrangian because this computational restriction has been removed. They found problems with their implementation of semi-Lagrangian advection leading to excessive eddy kinetic energy. This problem has been traced to not using positive definite interpolation for the advection of momentum, Hortal [9], see Ritchie et al [16] for full details of their implementation.

NCAR have been working on a semi-Lagrangian version of their community climate model and hope to make this the default in late 1994/early 1995. The problems they originally found with their model were due to using second order accurate (linear) interpolation at low resolution. The use of fourth order accurate (cubic) interpolation removed these problems. They have found it necessary to retune their cloud scheme as the moisture transport has changed near the tropopause and may still have to use a non-interpolating scheme in the vertical. This is not surprising as we expect the semi-Lagrangian transport of moisture in the vertical to be fourth order accurate not first order accurate and some retuning of the physics is likely to be required in areas where the moisture field has strong gradients. This extra accuracy of the advection may also lead to a change in the basic circulation around the tropopause and may uncover inadequacies in our understanding and treatment of the dynamical and physical processes in this area. Certainly it is the area in the model where both NCAR and ECMWF have found the most trouble in getting semi-Lagrangian advection to produce similar results to their Eulerian models. It is possible that to get the benefits possible from the semi-Lagrangian scheme is necessary to improve other parts of the model. As to what these other parts are, and whether they are deficient in the Unified Model, remains to be shown. Conservation is enforced in an a posteriori manner by adding a constant to each field to maintain the initial integral of the quantity. This procedure is performed every so many timesteps with the user chosing how often. See Williamson and Olson [19] for details.

Golding [7] developed a semi-Lagrangian advection scheme for the old non–hydrostatic mesoscale model. He showed that with this version of the model he was able to eliminate diffusion from the model. The ability to run a semi-Lagrangian semi-implicit model with no diffusion has also been found by Bates et al [1],[2]. Malcolm [11] has shown that the diffusion coefficients used in the Uni-

fied Model are remarkably similar to those which give the best answer, in terms of root mean square error, for the advection of the cosine bell on the sphere under solid body rotation. It is highly likely that the diffusion in the current Unified Model is purely there because of this and that no diffusion will be required in the semi-Lagrangian version of the proposed semi-implicit integration scheme for the Unified Model. It is thus proposed that there will no diffusion scheme in the semi-Lagrangian version of the the semi-implicit Unified Model and thus all the problems associated with these high impact, highly tunable and low physically realistic scheme will be removed.

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Figure 1: Initial Data.

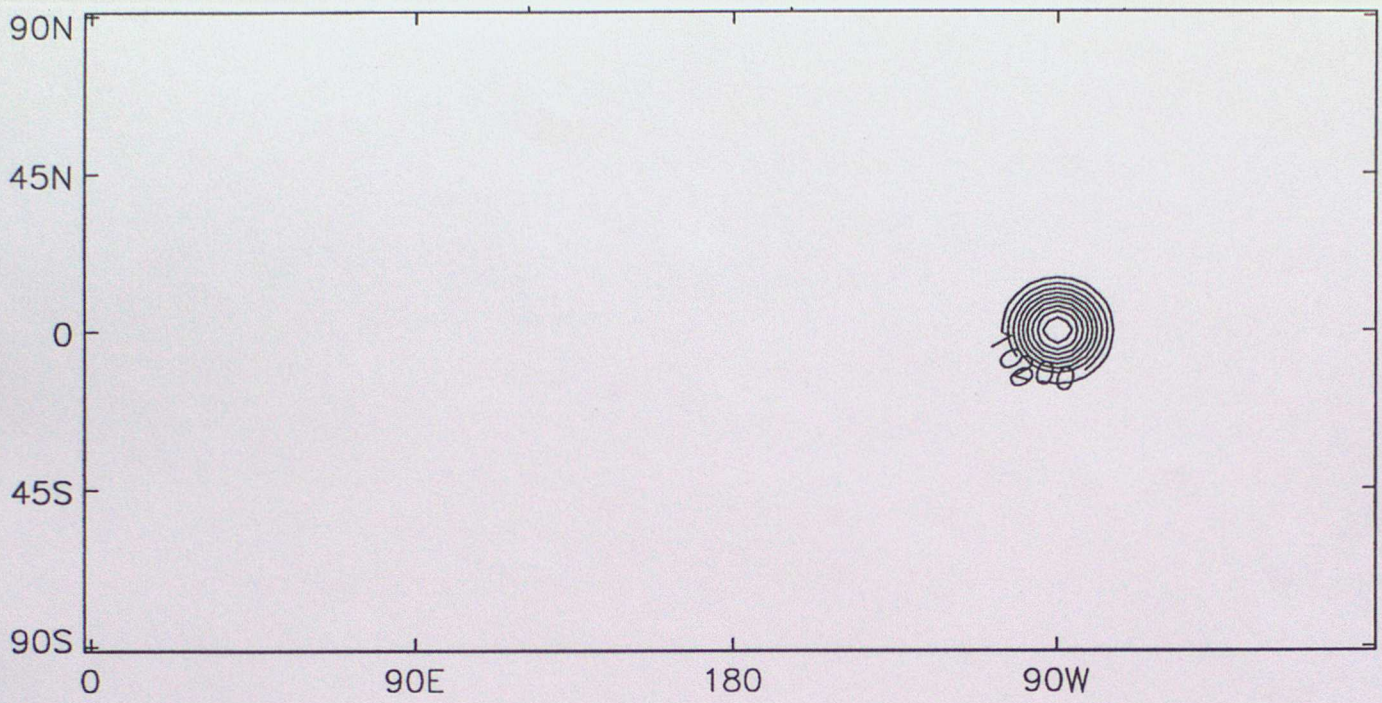

Figure 2: Tracer Advection scheme after 6 days.

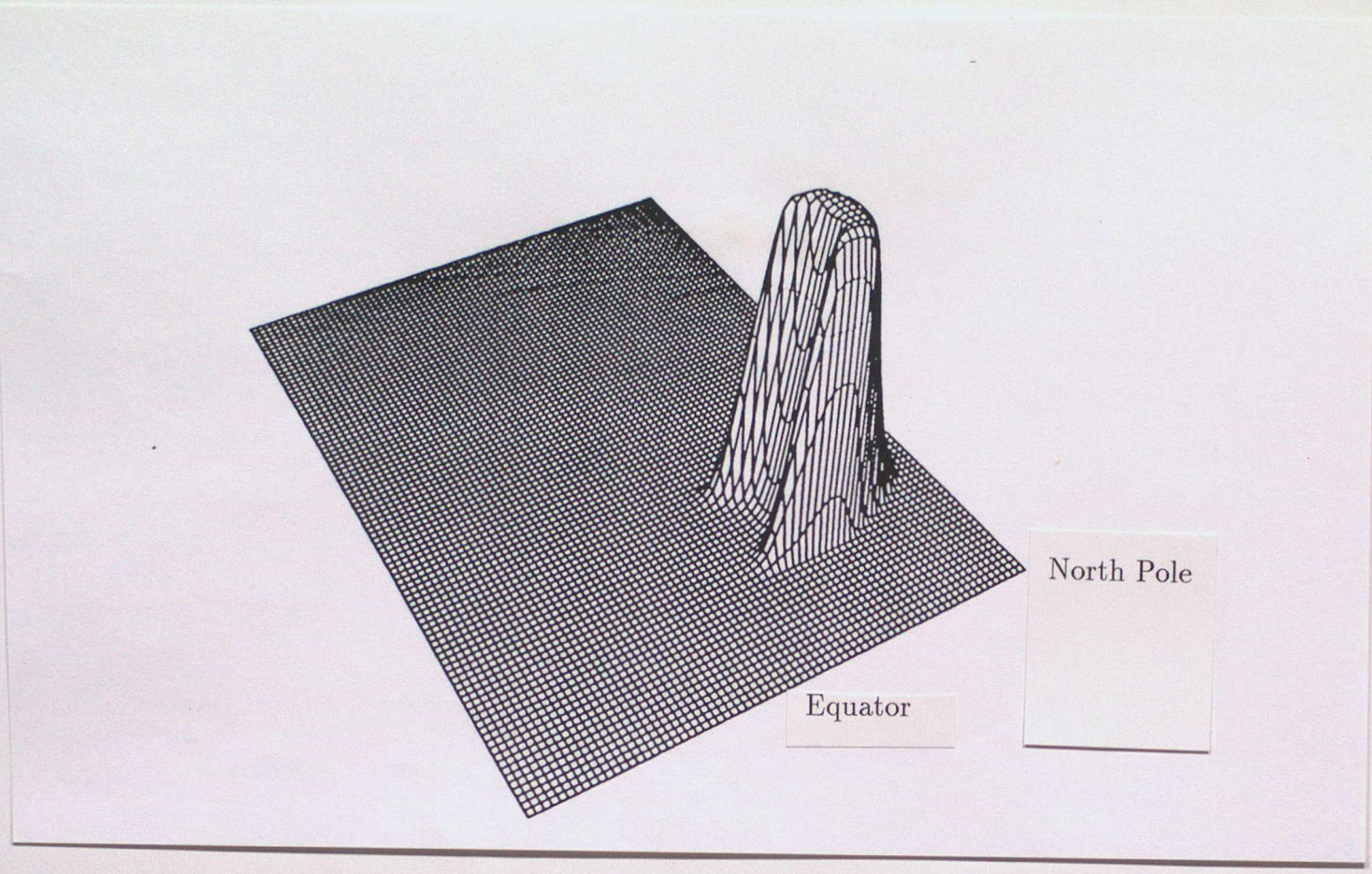
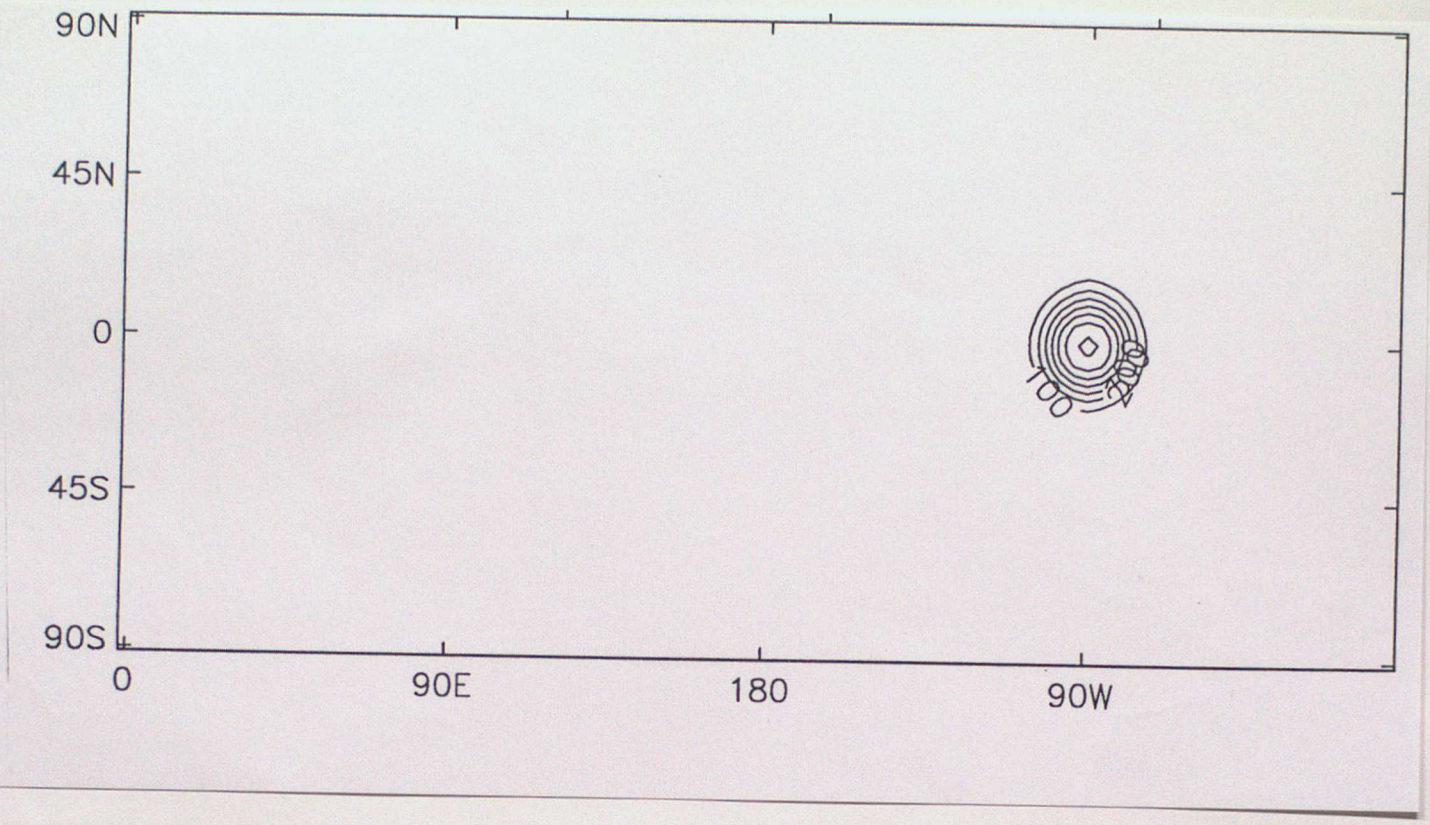
Figure 3: Monotone semi-Lagrangian scheme after 12 days.